

ON EFFECTS OF STOCHASTIC REGULARIZATION FOR THE PRESSURELESS GAS DYNAMICS

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ABSTRACT. We extend our result of [1] and show that one can associate with the stochastically perturbed non-viscid Burgers equation a system of viscous balance laws. The Cauchy data for the Burgers equation generates the data for this system. Till the moment of the shock formation in the solution to the Burgers equation the above system of viscous balance laws can be reduced to the pressureless gas dynamics system (in a limit as the parameters of perturbation tend to zero). If the solution to the Burgers equation contains shocks, the limit system is equivalent to the system with a specific pressure, in some sense analogous to the pressure of barotropic monoatomic gas.

1. STOCHASTIC PERTURBATION OF THE BURGERS EQUATION

Let us consider the Cauchy problem for the non-viscous Burgers equation:

$$(1.1) \quad \partial_t u + (u, \nabla)u = 0, \quad t > 0, \quad u(x, 0) = u_0(x) \in C^1(\mathbb{R}^n) \cap C_b(\mathbb{R}^n).$$

Here $u(x, t) = (u_1, \dots, u_n)(x, t)$ is a vector-function $\mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$.

We associate with (1.1) the following system of stochastic differential equations:

$$(1.2) \quad \begin{aligned} dX_k(t) &= U_k(t)dt + \sigma_1 d(W_k^1)_t, & dU_k(t) &= \sigma_2 d(W_k^2)_t, & k &= 1, \dots, n, \\ X(0) &= x, & U(0) &= u, & t &> 0, \end{aligned}$$

where $X(t)$ and $U(t)$ are considered as random variables with given initial distributions, $(X(t), U(t))$ runs in the phase space $\mathbb{R}^n \times \mathbb{R}^n$, σ_1 and σ_2 are nonnegative constants such that $|\sigma| \neq 0$ ($\sigma = (\sigma_1, \sigma_2)$)

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and $(W^i)_t = (W_1^i, \dots, W_n^i)_t$, $i = 1, 2$, are independent n - dimensional Brownian motions.

Let $P(t, x, u)$ be a probability density of joint distribution of random values (X, U) , subject to initial data

$$(1.3) \quad P_0(x, u) = \delta(u - u_0(x))\rho_0(x) = \prod_{k=1}^n \delta(u_k - (u_0(x))_k)\rho_0(x),$$

where $\rho_0(x)$ is a bounded nonnegative function from $L_{loc}^1(\mathbb{R}^n)$.

Let us introduce functions

$$(1.4) \quad \rho_\sigma(t, x) = \int_{\mathbb{R}^n} P(t, x, u) du, \quad u_\sigma(t, x) = \frac{\int_{\mathbb{R}^n} u P(t, x, u) du}{\int_{\mathbb{R}^n} P(t, x, u) du}.$$

Notice that $u_\sigma(0, x) = u_0(x)$. Certain properties of $u_\sigma(t, x)$ have been established in [2] and [3].

The density $P = P(t, x, u)$ obeys the Fokker-Planck equation

$$(1.5) \quad \frac{\partial P}{\partial t} = \left[- \sum_{k=1}^n u_k \frac{\partial}{\partial x_k} + \sum_{k=1}^n \frac{1}{2} \sigma_1^2 \frac{\partial^2}{\partial x_k^2} + \sum_{k=1}^n \frac{1}{2} \sigma_2^2 \frac{\partial^2}{\partial u_k^2} \right] P,$$

subject to initial data (1.3).

We apply the Fourier transform with respect to variables x and u in (1.5), (1.3) and obtain the following Cauchy problem for the Fourier transform $\tilde{P} = \tilde{P}(t, \lambda, \xi)$ of $P(t, x, u)$:

$$(1.6) \quad \frac{\partial \tilde{P}}{\partial t} = -\frac{1}{2}(\sigma_1^2 |\lambda|^2 + \sigma_2^2 |\xi|^2) \tilde{P} + (\lambda, \frac{\partial \tilde{P}}{\partial \xi}),$$

$$(1.7) \quad \tilde{P}(0, \lambda, \xi) = \int_{\mathbb{R}^n} e^{-i(\lambda, s)} e^{-i(\xi, u_0(s))} \rho_0(s) ds, \quad \lambda, \xi \in \mathbb{R}^n.$$

Equation (1.6) can easily be integrated and we obtain the solution:

$$(1.8) \quad \tilde{P}(t, \lambda, \xi) = \tilde{P}(0, \lambda, \xi + \lambda t) \exp \left(-\frac{1}{2} \sigma_1^2 |\lambda|^2 t - \frac{\sigma_2^2 t}{6} (|\lambda|^2 t^2 + 3t(\lambda, \xi) + 3|\xi|^2) \right).$$

The inverse Fourier transform (in the distributional sense) allows to find the density $P(t, x, u)$:

$$P(t, x, u) = \frac{1}{(2\pi)^{2n}} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} e^{i(\lambda, x)} e^{i(\xi, u)} \tilde{P}(t, \lambda, \xi) d\lambda d\xi =$$

$$\begin{aligned}
&= \frac{1}{(2\pi)^{2n}} \int_{\mathbb{R}^n} \rho_0(s) \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} e^{-\frac{1}{2}\Lambda^2} e^{-\frac{1}{2}\Xi^2} e^{-\frac{|u_0(s)-u|^2}{2t\sigma_1^2} - \frac{6|(u_0(s)+u)\frac{t}{2}+s-x|^2}{12\sigma_1^2 t + \sigma_2^2 t^3}} d\lambda d\xi ds = \\
(1.9) \quad &= C \int_{\mathbb{R}^n} \rho_0(s) e^{-\frac{|u_0(s)-u|^2}{2t\sigma_1^2} - \frac{6|(u_0(s)+u)\frac{t}{2}+s-x|^2}{12\sigma_1^2 t + \sigma_2^2 t^3}} ds,
\end{aligned}$$

where

$$\begin{aligned}
\Lambda &= \Lambda(t, \lambda, s) = \frac{\sqrt{12\sigma_1^2 t + \sigma_2^2 t^3}}{2\sqrt{3}} \lambda + \frac{2i\sqrt{3}}{\sqrt{12\sigma_1^2 t + \sigma_2^2 t^3}} ((u_0(s)+u)\frac{t}{2} + s - x), \\
\Xi &= \Xi(t, \xi, s) = \sigma_2 \sqrt{t} \xi + \frac{1}{\sigma_2 \sqrt{t}} \left(\frac{\sigma_2^2}{2} \lambda t^2 + i u_0(s) - u \right), \quad C = \left(\frac{\sqrt{3}}{\pi \sigma_2 t \sqrt{12\sigma_1^2 + \sigma_2^2 t^2}} \right)^n.
\end{aligned}$$

From (1.9) we have

$$(1.10) \quad \rho_\sigma(t, x) = \left(\frac{\sqrt{3}}{(\sqrt{2\pi t(3\sigma_1^2 + \sigma_2^2 t^2)})} \right)^n \int_{\mathbb{R}^n} \rho_0(s) e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2 + \sigma_2^2 t^2)}} ds,$$

$$(1.11) \quad u_\sigma(t, x) = \frac{1}{2(3\sigma_1^2 + \sigma_2^2 t^2)} \frac{\int_{\mathbb{R}^n} F(t, x, s) \rho_0(s) e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2 + \sigma_2^2 t^2)}} ds}{\int_{\mathbb{R}^n} \rho_0(s) e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2 + \sigma_2^2 t^2)}} ds},$$

where $F(t, x, s) = 6\sigma_1^2 u_0(s) - \sigma_2^2 t(tu_0(s) + 3(s-x))$.

2. PROPERTIES OF THE “PERTURBED VELOCITY”

Proposition 2.1. *Let u_0 and $\rho_0 > 0$ be functions of class $C^1(\mathbb{R}^n) \cap C_b(\mathbb{R}^n)$. If $t_*(u_0) > 0$ is a moment of time such that the solution to the Cauchy problem (1.1) with the initial condition u_0 keeps this smoothness for $0 < t < t_*(u_0) \leq +\infty$, then $\hat{u}_\sigma(t, x)$ tends to a solution of problem (1.1) as $|\sigma| \rightarrow 0$ for any fixed $(t, x) \in \mathbb{R}^{n+1}$, $0 < t < t_*(u_0)$.*

Proof. Let us denote by $J(u_0(x))$ the Jacobian matrix of the map $x \mapsto u_0(x)$. As it was shown in [4] (Theorem 1), if $J(u_0(x))$ has at least one eigenvalue which is negative for a certain point $x \in \mathbb{R}^n$, then the classical solution to (1.1) fails to exist beyond a positive time $t_*(u_0)$. Otherwise, $t_*(u_0) = \infty$. The matrix $C(t, x) = (I + tJ(u_0(x)))$, where I is the identity matrix, fails to be invertible for $t = t_*(u_0)$.

From (1.11) and (1.10) we have

$$u_\sigma(t, x) \rho_\sigma(t, x) = (u_\sigma^I(t, x) - u_\sigma^{II}(t, x)) \rho_\sigma(t, x),$$

where

$$u_\sigma^I(t, x) = K \int_{\mathbb{R}^n} u_0(s) \rho_0(s) e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2+\sigma_2^2t^2)}} ds,$$

$$u_\sigma^{II}(t, x) = \frac{3K\sigma_2^2t}{2(3\sigma_1^2+\sigma_2^2t^2)} \int_{\mathbb{R}^n} (tu_0(s) + s - x) \rho_0(s) e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2+\sigma_2^2t^2)}} ds,$$

$$K = \frac{\sqrt{3}}{\sqrt{2\pi t(3\sigma_1^2+\sigma_2^2t^2)}}.$$

Using the weak convergence of measures and the fact that ρ_0 and u_0 are continuous and bounded, we get for $u_\sigma^I(t, x)$:

$$\begin{aligned} \lim_{|\sigma| \rightarrow 0} u_\sigma^I(t, x) &= \frac{\int_{\mathbb{R}^n} u_0(s) \rho_0(s) \lim_{\substack{\sigma_1 \rightarrow 0 \\ \sigma_2 \rightarrow 0}} \left(\frac{\sqrt{3}}{\sqrt{2\pi t(3\sigma_1^2+\sigma_2^2t^2)}} \right)^n e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2+\sigma_2^2t^2)}} ds}{\int_{\mathbb{R}^n} \rho_0(s) \lim_{\substack{\sigma_1 \rightarrow 0 \\ \sigma_2 \rightarrow 0}} \left(\frac{\sqrt{3}}{\sqrt{2\pi t(3\sigma_1^2+\sigma_2^2t^2)}} \right)^n e^{-\frac{3|u_0(s)t+s-x|^2}{2t(3\sigma_1^2+\sigma_2^2t^2)}} ds} = \\ &= \frac{\int_{\mathbb{R}^n} u_0(s) \rho_0(s) \delta_{p(t,x,s)} ds}{\int_{\mathbb{R}^n} \rho_0(s) \delta_{p(t,x,s)} ds}, \end{aligned}$$

with $p(t, x, s) = u_0(s)t + s - x$, where δ_y is the Dirac measure as $y \in \mathbb{R}^n$. We can then on the basis on the invertibility of $C(t, x)$ use locally the implicit function theorem and find $s = s_{t,x}(p)$.

Therefore,

$$\begin{aligned} \lim_{|\sigma| \rightarrow 0} u_\sigma^I(t, x) &= \frac{\int_{\mathbb{R}^n} u_0(s_{t,x}(p)) \rho_0(s_{t,x}(p)) \det(C(t, s_{t,x}(p)))^{-1} \delta_p(ds_{t,x})}{\int_{\mathbb{R}^n} \rho_0(s_{t,x}(p)) \det(C(t, s_{t,x}(p)))^{-1} \delta_p(ds_{t,x})} = \\ &= u_0(s_{t,x}(0)). \end{aligned}$$

Now we prove that $\lim_{|\sigma| \rightarrow 0} u_\sigma^{II}(t, x) = 0$. We change variable $\tilde{s} = \frac{\sqrt{3}(u_0(s)t+s-x)}{\sqrt{t(3\sigma_1^2+\sigma_2^2t^2)}}$ and obtain

$$\lim_{|\sigma| \rightarrow 0} u_\sigma^{II}(t, x) = \lim_{|\sigma| \rightarrow 0} \frac{\int_{\mathbb{R}^n} \sigma_2^2 t \rho_0(\tilde{s}) \exp(-\tilde{s}^2/2) d\tilde{s}}{\rho_\sigma(t, x)} = 0.$$

Let us introduce the new notation $s_0(t, x) \equiv s_{t,x}(0)$. Then the following vectorial equation holds:

$$(2.1) \quad u_0(s_0(t, x))t + s_0(t, x) - x = 0.$$

The function $u(t, x) = u_0(s_0(t, x))$ satisfies the Burgers equation. To check this fact it is enough to differentiate (2.1) with respect to t and

x_j and find $s_{0,t}$ and $s_{0,x}$. The more detailed proof you can find in [1] in Proposition 1. \square

It is important to note that $s_0(t, x)$ is unique for all t for which the solution to the Burgers equation $u(t, x)$ is smooth.

Remark 2.2. Proposition 2.1 can naturally be extended to the class of functions ρ_0 such that there exists a sequence $\rho_0^\varepsilon \in C^1(\mathbb{R}^n) \cap C_b(\mathbb{R}^n)$ converging to ρ_0 as $\varepsilon \rightarrow 0$ almost everywhere. In this case $u_\sigma(t, x)$ tends to a solution of problem (1.1) as $\sigma \rightarrow 0$ almost everywhere on $(t, x) \in \mathbb{R}^{n+1}$, $0 < t < t_*(u_0)$.

Theorem 2.3. *The scalar function $\rho_\sigma(t, x)$ and the vector-function $u_\sigma(t, x)$, defined in (1.4), solve the following system:*

$$(2.2) \quad \frac{\partial \rho_\sigma}{\partial t} + \operatorname{div}_x(\rho_\sigma u_\sigma) = \frac{1}{2} \sigma_1^2 \sum_{k=1}^n \frac{\partial^2 \rho_\sigma}{\partial x_k^2},$$

$$(2.3) \quad \frac{\partial(\rho_\sigma u_{\sigma,i})}{\partial t} + \operatorname{div}_x(\rho_\sigma u_{\sigma,i} u_\sigma) = \frac{1}{2} \sigma_1^2 \sum_{k=1}^n \frac{\partial^2(\rho_\sigma u_{\sigma,i})}{\partial x_k^2} - I_\sigma$$

where $i = 1, \dots, n$, $t \geq 0$ and $I_\sigma = \int_{\mathbb{R}^n} (u_i - u_{\sigma,i})((u - u_\sigma), \nabla_x P(t, x, u)) du$.

Proof. Integrating (1.5) with respect of u we get:

$$\frac{\partial \rho_\sigma}{\partial t} + \operatorname{div}_x(\rho_\sigma u_\sigma) = \frac{1}{2} \sigma_1^2 \sum_{k=1}^n \frac{\partial^2 \rho_\sigma}{\partial x_k^2} + \frac{1}{2} \sigma_2^2 \sum_{k=1}^n \int_{\mathbb{R}^n} \frac{\partial^2 P(t, x, u)}{\partial u_k^2} du.$$

The last integral is equal to zero, because $P(t, x, u)$ and all its derivatives tend to zero as $|u| \rightarrow \infty$. Thus, we obtain (2.2).

To prove (2.3) we note that the definitions of $u_\sigma(t, x)$ and $\rho_\sigma(t, x)$ imply

$$(2.4) \quad \begin{aligned} \frac{\partial(\rho_\sigma u_{\sigma,i})}{\partial t} &= \frac{\partial}{\partial t} \int_{\mathbb{R}^n} u_i P du = \int_{\mathbb{R}^n} u_i P_t du = - \int_{\mathbb{R}^n} u_i (u, \nabla_x P) du + \\ &+ \frac{1}{2} \sigma_1^2 \sum_{k=1}^n \int_{\mathbb{R}^n} u_i \frac{\partial^2 P}{\partial x_k^2} du + \frac{1}{2} \sigma_2^2 \sum_{k=1}^n \int_{\mathbb{R}^n} u_i \frac{\partial^2 P}{\partial u_k^2} du, \end{aligned}$$

where $P_t \equiv \frac{\partial}{\partial t} P$. Early we noted that the last integral is equal to zero.

Further, for $i = 1, \dots, n$ we have

$$\operatorname{div}_x(\rho u_{\sigma,i} u_\sigma) = u_{\sigma,i} \operatorname{div}_x(\rho_\sigma u_\sigma) + u_\sigma \partial_x(\rho_\sigma u_{\sigma,i}) - u_\sigma u_{\sigma,i} \partial_x \rho_\sigma =$$

$$(2.5) \quad = \int_{\mathbb{R}^n} u_{\sigma,i}(u, \nabla_x P) du + \int_{\mathbb{R}^n} u_i(u_\sigma, \nabla_x P) du - \int_{\mathbb{R}^n} u_{\sigma,i}(u_\sigma, \nabla_x P) du,$$

where $i, k = 1, \dots, n$. Equation (2.3) follows immediately from (2.4) and (2.5). \square

Let us set $\rho(t, x) = \lim_{|\sigma| \rightarrow 0} \rho_\sigma(t, x)$ and $\bar{u}(t, x) = \lim_{|\sigma| \rightarrow 0} u_\sigma(t, x)$.

Theorem 2.4. *Assume that $(\rho(t, x), \bar{u}(t, x))$, the limits of (ρ_σ, u_σ) as $|\sigma| \rightarrow 0$, are C^1 - smooth bounded functions for $(t, x) \in \Omega := [0, t_*(u_0)) \times \mathbb{R}^n$, $t_*(u_0) \leq \infty$. Then they solve in Ω the pressureless gas dynamics system*

$$(2.6) \quad \partial_t \rho + \operatorname{div}_x(\rho \bar{u}) = 0, \quad \partial_t(\rho \bar{u}) + \nabla_x(\rho \bar{u} \otimes \bar{u}) = 0.$$

Proof. As follows from Proposition 2.1, the function $\bar{u}(t, x)$ is a C^1 - solution of the non-viscous Burgers equation. Further, (2.2) is a linear parabolic equation with respect to ρ_σ , hence the limit as $|\sigma| \rightarrow 0$ reduces it to the first equation in (2.6) (continuity equation). The second equation in (2.6) is a corollary of the non-viscous Burgers equation and continuity equation for smooth solutions. \square

Remark 2.5. Proposition 2.4 implies that the integral term on the right-hand side of (2.3) vanishes as $|\sigma| \rightarrow 0$ in the case of smooth limit functions ρ and \bar{u} .

3. A REGULARIZATION OF THE PRESSURELESS GAS DYNAMICS

After the moment of shock formation in the solution to the Burgers equation the limit of (2.2)-(2.3) as $|\sigma| \rightarrow 0$ is not equivalent to the pressureless gas dynamics system (2.6), the integral term in (2.3) does not vanish and can be interpreted as a gradient of special pressure (spurious pressure). It is well known that for the pressureless gas dynamic system after the moment of singularity formation there arise a question how to define a generalized solution, and the answer depends on the prescribed type of interaction between particles. For the sticky particles model, where two particles move together after interaction, one need to introduce a strongly singular solution with a delta-singularity in the density component. If we assume that particles do not interact, we get the generalized solution in the sense of free particles, whose singularities are only shocks. In this sense we can call the limit of system (2.2)-(2.3) as $|\sigma| \rightarrow 0$ a regularization of the pressureless gas dynamics equations. There exist an algorithm how to pass from the model of free particles to the model of sticky particles (see [1] for details in the case $\sigma_2 = 0$).

For discontinuous initial data analogously to [1] we introduce a generalized solution in the sense of free particles:

Definition 3.1. We call the couple of functions $(\rho_{FP}(t, x), u_{FP}(t, x))$ the generalized solution to the Cauchy problem for (2.6) in the sense of free particles (FP-generalized solution) subject to initial data $(\rho_0(x), u_0(x)) \in \mathbb{L}_{loc}^2(\mathbb{R}^n) \cap \mathbb{L}_\infty(\mathbb{R}^n)$, if for almost all $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$

$$\rho_{FP}(t, x) = \lim_{\varepsilon \rightarrow 0} (\lim_{\sigma \rightarrow 0} \rho_\sigma^\varepsilon(t, x)), \quad u_{FP}(t, x) = \lim_{\varepsilon \rightarrow 0} (\lim_{\sigma \rightarrow 0} u_\sigma^\varepsilon(t, x)),$$

where $(\rho_\sigma^\varepsilon(t, x), u_\sigma^\varepsilon(t, x))$ correspond to initial data $\rho_0^\varepsilon = \eta_\varepsilon * \rho_0$, $u_0^\varepsilon = \eta_\varepsilon * u_0$, where $\eta_\varepsilon(x)$ is the standard averaging kernel.

In [1] for $\sigma_2 = 0$ we solve the Riemann problem in the sense of free particles, this can be done analogously in the general case $\sigma_2 > 0$.

It is very important that the double limit procedure used in the definition of the FP-solution plays role only for the case of a central rarefaction wave, where it helps to find a unique stable solution in the vacuum domain (we refer to [1] again). For the compression wave the FP-solution can be computed directly by formulae (1.10), (1.11) using only one limit pass $|\sigma| \rightarrow 0$. Thus, after the moment of the shock formation $t_*(u_0)$ in the Burgers equation the solution to the respective conservation law

$$(3.1) \quad \partial_t \rho + \text{div}_x(\rho \bar{u}) = 0, \quad \partial_t(\rho \bar{u}) + \nabla_x(\rho \bar{u} \otimes \bar{u}) + \nabla p = 0, \quad \nabla p = \lim_{|\sigma| \rightarrow 0} I_\sigma$$

can be found from (1.10), (1.11) as $|\sigma| \rightarrow 0$. By definition the solution to (3.1) is the FP-solution to (2.6), thus (3.1) can be considered as a regularization of (2.6).

4. THE 1D MODEL FOR POLYTROPIC GAS

Let us *assume* that the gas that obeys system (3.1) is polytropic, i.e. the state equation is $p = A\rho^\gamma$, γ is the adiabatic exponent, A is a positive constant. Therefore we have to supplement (3.1) with equation

$$\partial_t p + u \partial_x p + \gamma p \partial_x u = 0.$$

We consider the 1D Riemann problem in the compression case $u_2 < 0$: $\rho_0(x) = \rho_1 + \rho_2 \theta(x)$, $u_0(x) = u_1 + u_2 \theta(x)$, where ρ_i , u_i , $i = 1, 2$ are constants, In [1] the solution was obtained (for $\sigma_2 = 0$, the general case is analogous):

$$\begin{aligned} \rho(t, x) &= \rho_1 + (\rho_1 + \rho_2) \theta(x - u_1 t) - \rho_1 \theta(x - (u_1 + u_2) t), \\ u(t, x) &= u_1 + \frac{\rho_1 + \rho_2}{2\rho_1 + \rho_2} u_2 \theta(x - u_1 t) - \frac{\rho_1}{2\rho_1 + \rho_2} u_2 \theta(x - (u_1 + u_2) t), \end{aligned}$$

$$p(t, x) = \frac{\rho_1(\rho_1 + \rho_2)}{2\rho_1 + \rho_2} u_2^2 (\theta(x - (u_1 + u_2)t) - \theta(x - u_1 t)).$$

Proposition 4.1. *In assumption that the state equation is polytropic, the adiabatic exponent $\gamma \rightarrow 3$ as the ratio $\frac{\rho_1}{\rho_2} \rightarrow 0$.*

Proof. We have three Hugoniot conditions, that follow from the mass conservation, momentum conservation ((3.1)) and the conservation of total energy $E = \frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1}$. We denote by f_- and f_+ the values of $f(x)$ before and after shock, respectively. For the shock moving with the speed $D = u_1 + u_2$ we have $\rho_+ = 2\rho_1 + \rho_2$, $\rho_- = \rho_1$, $u_+ = u_1 + \frac{\rho_1 + \rho_2}{2\rho_1 + \rho_2} u_2$, $u_- = u_1$, $p_+ = \frac{\rho_1(\rho_1 + \rho_2)}{2\rho_1 + \rho_2} u_2^2$, $p_- = 0$. The third Hugoniot condition gives $D = \frac{[Eu + pu]}{[E]}$ and $\gamma = \frac{3\rho_1 + \rho_2}{\rho_1 + \rho_2}$. This implies the proposition. The consideration of the second shock gives the same result. \square

The adiabatic exponent $\gamma = 1 + \frac{2}{n}$, where n is the space dimension, corresponds to the monoatomic gas.

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